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Short-range correlations and neutrinoless double beta decay

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Abstract

In this work we report on the effects of short-range correlations upon the matrix elements of neutrinoless double beta decay ($0\nu\beta\beta$). We focus on the calculation of the matrix elements of the neutrino-mass mode of $0\nu\beta\beta$ decays of ^{48}Ca and ^{76}Ge. The nuclear-structure components of the calculation, that is the participant nuclear wave functions, have been calculated in the shell-model scheme for ^{48}Ca and in the proton–neutron quasiparticle random-phase approximation (pnQRPA) scheme for ^{76}Ge. We compare the traditional approach of using the Jastrow correlation function with the more complete scheme of the unitary correlation operator method (UCOM). Our results indicate that the Jastrow method vastly exaggerates the effects of short-range correlations on the $0\nu\beta\beta$ nuclear matrix elements.

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Keywords: Neutrinoless double beta decay; Nuclear matrix elements; Short-range correlations; Unitary correlation operator method

A good knowledge of the nuclear matrix elements governing the decay rates of neutrinoless double beta ($0\nu\beta\beta$) decay is mandatory if one wants to extract information about the neutrino mass from the current experimental limits for the half-lives of $0\nu\beta\beta$-decay transitions [1,2]. The standard theoretical methods which are suitable for the calculation of the relevant nuclear matrix elements can be found in the literature, e.g. in [3,4]. Although many of the formalisms are well established, difficulties arise from the approximations which are needed in order to perform the actual calculations.

In the mass mode of the $0\nu\beta\beta$ decay the involved two nucleons exchange a light Majorana neutrino [5]. Average value of the exchanged momentum is of the order of 100–200 MeV$/c$ and thus the involved nucleons are on average at close distance from each other. There is, however, a minimum relative distance of the order of 1 fm after which the two nucleons may eventually overlap. In nuclear matter this overlapping cannot happen and in theoretical description of the $0\nu\beta\beta$ decay one needs to take into account this fact. Based on this it has been argued [6] that special measures have to be taken when performing nuclear-structure calculations using the mean-field picture with residual two-body interactions between the two interacting nucleons. In the case of the $0\nu\beta\beta$ decay these measures boil down to introducing an explicit Jastrow type of correlation function into the involved two-body transition matrix elements [3]. Using this method in the numerical calculations of $0\nu\beta\beta$-decay matrix elements considerable corrections to the involved Fermi and Gamow–Teller nuclear matrix elements were reported [3,7].

In [8] a different method was used to explicitly take into account the short-range correlations. This approach is based on the use of the Horie–Sasaki method to evaluate the involved radial form factors and the short-range correlations were considered to arise from the ω exchange in the nucleon–nucleon interaction [9,10]. Contrary to [3,7], in this approach only relatively small corrections to the involved nuclear matrix elements were obtained [8,11]. Instead of using the above described methods, one can use the more complete new concept of unitary correlation operator method (UCOM) [12] to take into account the short-range effects in $0\nu\beta\beta$ decay. In this method a unitary cor-

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relation operator moves a pair of nucleons away from each other whenever they start to overlap. This method also conserves the probability normalization of the relative wave function.

In this work we address the important issue of short-range correlations in the computation of nuclear matrix elements involved in the neutrinoless double beta decay. We have used both the UCOM and Jastrow methods and we compare the results for the ground-state-to-ground-state $0\nu\beta\beta$ decays of ^{48}Ca and ^{76}Ge. For ^{48}Ca we calculate the relevant nuclear wave functions in the solid theoretical framework of the nuclear shell model. In order to accomplish this we have used the OXBASH code [13], which is actually available to any practitioner in the field. For ^{76}Ge we have used the framework of the proton–neutron quasiparticle random-phase approximation (pnQRPA), suitable for calculations of nuclear properties of medium-heavy and heavy nuclei.

Our calculated results show that the reduction caused by the inclusion of the short range correlations depends on the multipole which contributes to the $0\nu\beta\beta$ matrix element. In addition, the strength distributions of the multipoles are practically unaltered by the short-range correlations, suggesting the effect to be just an overall multipole-dependent scaling. This scaling factor is close to unity for all multipoles in the UCOM scheme, but acquires strongly reduced values for high multipoles in case of the Jastrow method.

We start the quantitative scrutiny of the effects of short-range correlations by briefly presenting the central ideas behind our computations. By assuming the neutrino mass mechanism to be the dominant one in the $0\nu\beta\beta$ decay one can write as a good approximation the inverse of the half-life as [4]

$$\left[t_{1/2}^{(0\nu)}\right]^{-1} = G_1^{(0\nu)}\left(\frac{\langle m_\nu\rangle}{m_e}\right)^2\left(M_{\rm GT}^{(0\nu)} - \left(\frac{g_V}{g_A}\right)^2 M_{\rm F}^{(0\nu)}\right)^2, \quad (1)$$

where m_e is the mass of the electron and

$$\langle m_\nu\rangle = \sum_j \lambda_j^{\rm CP} m_j |U_{ej}|^2 \quad (2)$$

is the effective mass of the neutrino, $\lambda_j^{\rm CP}$ being the CP phase. Furthermore, the quantity $G_1^{(0\nu)}$ of Eq. (1) is the leptonic phase-space factor defined in [4]. The double Gamow–Teller and double Fermi nuclear matrix-elements in (1) are defined as [5]

$$M_{\rm F}^{(0\nu)} = \sum_a \left(0_f^+\left\|h_+(r_{mn}, E_a)\right\|0_i^+\right), \quad (3)$$

$$M_{\rm GT}^{(0\nu)} = \sum_a \left(0_f^+\left\|h_+(r_{mn}, E_a)(\sigma_m\cdot\sigma_n)\right\|0_i^+\right). \quad (4)$$

Here the summation runs over all states a of the intermediate nucleus, which in this case are ^{48}Sc and ^{76}As. The definition of the neutrino potential $h_+(r_{mn}, E_a)$ can be found in Refs. [3–5].

The traditional way [3] to include the short-range correlations in the $0\nu\beta\beta$ matrix elements is by introducing the Jastrow correlation function $f_{\rm J}(r)$. It has to be noted that this particular variant of the Jastrow function is a rudimentary one and does not do full justice to the name Jastrow correlations. For example, in light nuclei accurate Monte Carlo calculations are based on Jatrow-like correlations which are variationally determined and have different ansatz functions in different isospin channels. This fact notwithstanding we choose to call here the rudimentary approach of [3] as Jastrow method since this is the term adopted by the double-beta-decay community.

The Jastrow function depends on the relative distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$ of two nucleons, and in the Jastrow scheme one replaces the bare $0\nu\beta\beta$ operator $\mathcal{O}$ by a correlated operator $\mathcal{O}_{\rm J}$ by the simple procedure

$$\left(0_f^+\|\mathcal{O}\|0_i^+\right) \to \left(0_f^+\|\mathcal{O}_{\rm J}\|0_i^+\right) = \left(0_f^+\|f_{\rm J}\mathcal{O}f_{\rm J}\|0_i^+\right). \quad (5)$$

A typical choice for the function $f_{\rm J}$ is

$$f_{\rm J}(r) = 1 - e^{-ar}\left(1 - br^2\right), \quad (6)$$

with $a = 1.1$ fm^2 and $b = 0.68$ fm^2. As a result, the Jastrow function effectively cuts out the small r part from the relative wave function of the two nucleons. For this reason, the traditionally adopted Jastrow procedure does not conserve the norm of the relative wave function and one should use, in principle, the operator

$$\mathcal{O}_{\rm J} = \frac{f_{\rm J}\mathcal{O}f_{\rm J}}{\langle\Psi|f_{\rm J}f_{\rm J}|\Psi\rangle} \quad (7)$$

when the initial and final states are the same, here denoted by $|\Psi\rangle$. For different initial and final states normalization is even more problematic. Even a proper normalization does not guarantee that the short-range correlations would be correctly treated by the Jastrow procedure.

To circumvent the difficulties associated with the use of a Jastrow function one can adopt the more refined unitary correlation operator method (UCOM) [12] when discussing short-range effects in the $0\nu\beta\beta$ decay. In the UCOM one obtains the correlated many-particle state $|\tilde{\Psi}\rangle$ from the uncorrelated one as

$$|\tilde{\Psi}\rangle = C|\Psi\rangle, \quad (8)$$

where C is the unitary correlation operator. The operator C is a product of two unitary operators: $C = C_\Omega C_r$, where C_Ω describes short-range tensor correlations and C_r central correlations. Due to unitarity of the operator C, the norm of the correlated state is conserved. Moreover, one finds for the matrix element of an operator A

$$\langle\tilde{\Psi}|A|\tilde{\Psi}'\rangle = \langle\Psi|C^\dagger AC|\Psi'\rangle = \langle\Psi|\tilde{A}|\Psi'\rangle, \quad (9)$$

so that it is equivalent to use either correlated states or correlated operators.

The exact form of the operator C is gained by finding the minimum of the Hamiltonian matrix element $\langle\Psi|C^\dagger HC|\Psi\rangle$. Therefore, the choice of the two-body interaction in H affects also the form of C. Explicit expressions for the operators C_r and C_Ω can be found in Refs. [12,14]. Application of these expressions to the double Gamow–Teller and Fermi matrix elements shows that the tensor correlations of C_Ω vanish and we are left with only the central correlations.

For ^{48}Ca the nuclear-structure calculations were handled by the shell-model code OXBASH [13]. In our calculations we have used the FPBP two-body interaction of [15], which was

Table 1
Multipole decomposition and the total value of the matrix element $M_{\mathrm{GT}}^{(0\nu)}$ for ^{48}Ca. The cases are: no short-range correlations included (bare), with Jastrow correlations and with UCOM correlations using Bonn-A and Argonne V18 parametrizations

J^π	Bare	Jastrow	UCOM	
			Bonn-A	AV18
1^+	−0.330	−0.305	−0.322	−0.319
2^+	−0.117	−0.092	−0.108	−0.104
3^+	−0.327	−0.246	−0.302	−0.293
4^+	−0.066	−0.035	−0.054	−0.051
5^+	−0.246	−0.121	−0.212	−0.199
6^+	−0.042	−0.008	−0.030	−0.027
7^+	−0.150	−0.029	−0.120	−0.107
Sum	−1.278	−0.835	−1.150	−1.101

Table 2
The same as Table 1 but for $M_{\mathrm{F}}^{(0\nu)}$

J^π	Bare	Jastrow	UCOM	
			Bonn-A	AV18
1^+	0.000	0.000	0.000	0.000
2^+	0.185	0.145	0.174	0.169
3^+	0.000	0.000	−0.001	−0.001
4^+	0.116	0.061	0.102	0.096
5^+	0.000	0.000	−0.002	−0.002
6^+	0.061	0.012	0.050	0.045
7^+	0.000	0.000	−0.002	−0.002
Sum	0.367	0.221	0.324	0.308

obtained by fitting the Kuo–Brown interaction to experimental data. Due to the fact that we have limited our model space to the pf shell, the $0\nu\beta\beta$ matrix elements are composed of only positive-parity states. The shell-model calculations had to be truncated by requiring that the minimum number of particles in the $0f_{7/2}$ orbital be 4.

Our main results for ^{48}Ca are presented in Tables 1 and 2. In these tables we list the calculated multipole decomposition and total values of the matrix elements $M_{\mathrm{GT}}^{(0\nu)}$ and $M_{\mathrm{F}}^{(0\nu)}$ for four different cases. In the first case, which we refer to as bare matrix elements, we have not taken into account any short-range correlations. In the second case the short-range effects were handled by the use of the Jastrow function (6) and the replacement (5). In the third and fourth cases we have used the UCOM to account for the short-range effects. The Kuo–Brown interaction was not derived via UCOM, as it should be if it were to be used in the same calculation as the UCOM-derived double-beta operator. To access the magnitude of the resulting effect, we have adopted two different UCOM parameter sets in the present calculation. These two parameter sets were obtained by minimizing the energy for the Bonn-A and Argonne V18 potentials. Both of the used UCOM parameter sets can be found in [16].

As the results in Tables 1 and 2 indicate, the differences between the results obtained by the use of the two UCOM parameter sets are small. Therefore, we expect that the results obtained by the use of the Kuo–Brown UCOM parameters do not deviate significantly from the Bonn-A or Argonne V18 results. We also note that there exist a small UCOM contribution to the double Fermi matrix element $M_{\mathrm{F}}^{(0\nu)}$ coming from the odd-J intermediate states. This is explained by the fact that in Ref. [16] slightly different parameters were given to the $S=0$ and $S=1$ channels.

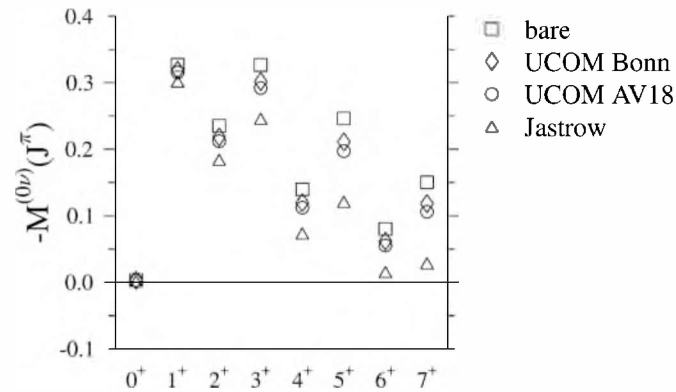

Fig. 1. Multipole decomposition of the total $0\nu\beta\beta$ decay matrix element $M_{\mathrm{GT}}^{(0\nu)} - (g_{\mathrm{V}}/g_{\mathrm{A}})^2 M_{\mathrm{F}}^{(0\nu)}$ for ^{48}Ca. The cases are: no short-range correlations included (bare), with Jastrow correlations and with UCOM correlations using the Bonn-A and Argonne V18 parametrizations.

In Fig. 1 we show graphically the multipole decomposition of the total matrix element $M_{\mathrm{GT}}^{(0\nu)} - (g_{\mathrm{V}}/g_{\mathrm{A}})^2 M_{\mathrm{F}}^{(0\nu)}$ of (1) for the four different cases of Tables 1 and 2. The ratio $g_{\mathrm{A}}/g_{\mathrm{V}} = -1.254$ was used in this plot. As can be seen, the results obtained by using the two different UCOM parameter sets do not differ significantly. Also, one can see that the effects of the Jastrow or UCOM correlations grow with increasing J of the intermediate states. For the extreme case of the 7^+ contributions the switching on of the Jastrow correlations changes the value of the matrix element $M_{\mathrm{GT}}^{(0\nu)}(7^+)$ from -0.150 to -0.029, roughly corresponding to a factor of 5 reduction. At the same time the UCOM correlations produce only a 20%–30% reduction from the bare matrix element. It seems that in a situation like this blind use of Jastrow correlations cuts out relevant parts of the nuclear many-body wave function. From the tables one deduces that the Jastrow correlations cause some 35%–40% reduction to the magnitudes of the total matrix elements, whereas the UCOM causes a reduction of 10%–16%. It is worth pointing out that our numbers for the Jastrow case coincide with the numbers of the corresponding earlier calculation performed by the Strasbourg group [17].

To trace the source of differences between the Jastrow and UCOM corrected matrix elements we show for the ^{48}Ca decay in Fig. 2 the radial dependence of the two-particle Gamow–Teller $0\nu\beta\beta$ matrix element in the special case of $p = p' = n = n' = 0f_{7/2}$ and $J = 7$ (this is the contribution to Eq. (4.16) of [3] without including the one-body transition densities and the overlap of the two complete sets of pnQRPA states). The oscillator parameter value $b = 2.0$ fm was used in the plot. For the case of UCOM contribution we have used the correlated wave functions and the approximation $r - R_-(r) \approx R_+(r) - r$ for illustrative purpose.[1] Thus, the UCOM plot should be taken only as a schematic one. From the figure one can see that the Jas-

[1] In all other numerical applications of the UCOM we have used correlated operators without involving any approximations.

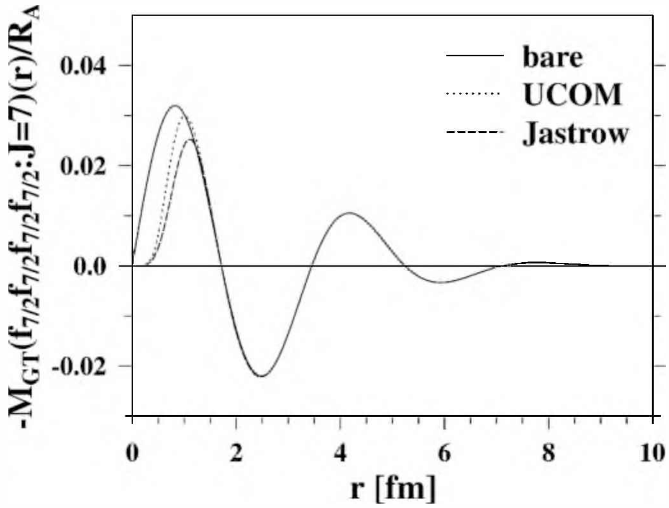

Fig. 2. Radial dependence of the two-particle Gamow–Teller $0\nu\beta\beta$ matrix element for $p = p' = n = n' = 0f_{7/2}$ and $J = 7$ in the case of ^{48}Ca decay. Shown are the bare matrix element and Jastrow and UCOM correlated matrix elements.

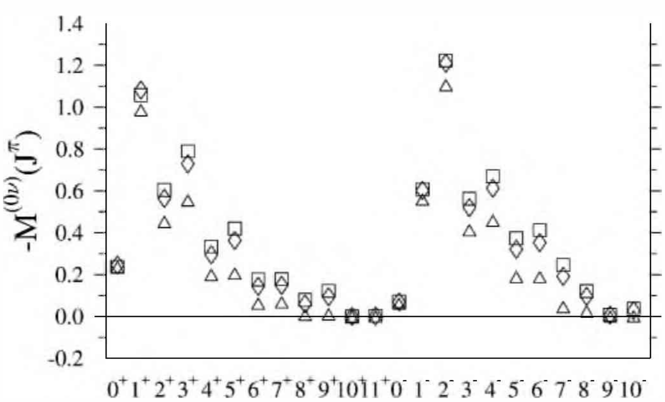

Fig. 3. The same as Fig. 1 for the decay of ^{76}Ge calculated by using the pnQRPA. Only the Bonn-A parametrization has been used for UCOM.

trow correlations cut out a significant part of the matrix element at small r. This leads to a situation, where the total integrated areas under the radial curve almost cancel out. In the case of UCOM correlations the cancellation is not as severe due to the fact that not so much amplitude is lost for small r.

Our results for the $0\nu\beta\beta$ decay of ^{76}Ge are summarized in Fig. 3 and Table 3. The results have been obtained by using the framework of the proton–neutron quasiparticle random-phase approximation (pnQRPA) [4,18]. The related calculations, including the BCS and the pnQRPA calculations for ^{76}Ge and ^{76}Se, were done in the model space consisting of the single-particle $1p$–$0f$–$2s$–$1d$–$0g$–$0h_{11/2}$ orbitals, both for protons and neutrons. The single-particle energies were obtained from a spherical Woods–Saxon potential. Slight adjustment was done for some of the energies at the vicinity of the proton and neutron Fermi surfaces to reproduce better the low-energy spectra of the neighboring odd-A nuclei and the low-energy spectrum of ^{76}As. The Bonn-A G-matrix [19] was used as a two-body interaction and it was renormalized in the standard way, as discussed e.g. in Refs. [20–22]. Due to this phenomenological renormalization we did not perform an additional UCOM renormalization of the two-body interaction. In the present calculations we have used the 'default value' $g_{pp} = 1.0$ for the particle–particle interaction parameter of the pnQRPA.

Table 3

Gamow–Teller ($M_{GT}^{(0\nu)}$), Fermi ($M_F^{(0\nu)}$) and total matrix elements derived from pnQRPA calculations for the $0\nu\beta\beta$ decay of ^{76}Ge. The cases 'bare', 'Jastrow' and 'UCOM' are as in Table 1. Only the Bonn-A parametrization has been used for UCOM

	Bare	Jastrow	UCOM Bonn-A
$M_{GT}^{(0\nu)}$	−6.755	−4.681	−6.265
$M_F^{(0\nu)}$	2.474	1.778	2.310
Total	−8.328	−5.811	−7.734

In Fig. 3 we display for ^{76}Ge decay the multipole decomposition of the total $0\nu\beta\beta$ matrix element $M_{GT}^{(0\nu)} - (g_V/g_A)^2 M_F^{(0\nu)}$ as derived from the pnQRPA calculations. The used symbols are the same as in Fig. 1. The ratio $g_V/g_A = -1.254$ was used in the calculations. Since the nuclear wave functions have been calculated by the use of the Bonn potential, we have used only the Bonn-A parametrization for the UCOM. Here one can see a pattern similar to the case of ^{48}Ca: the effect of the Jastrow correlations grows strongly with increasing value of the angular momentum of the intermediate states. As in the case of the ^{48}Ca decay the effect is the largest for the unnatural-parity states $1^+, 2^-, 3^+, 4^-, \ldots$ in an odd–odd nucleus. Contrary to the Jastrow-corrected multipole contributions, the UCOM-corrected ones stay close to the bare contributions for all intermediate multipoles J^π.

We summarize our results on the $0\nu\beta\beta$ matrix elements of the ^{76}Ge decay in Table 3, where we give the bare, Jastrow-corrected and UCOM-corrected Gamow–Teller, Fermi and total matrix elements. For the total matrix element the Jastrow corrections amount to 30% reduction from the bare matrix element, whereas the UCOM corrections are some 7%. This, again, suggests that in the earlier calculations [3,7] the effect of the short-range correlations has been considerably overestimated.

In this Letter we have addressed the important issue of short-range correlations in the context of neutrinoless double beta decay. We have calculated the related nuclear matrix by the nuclear shell model for ^{48}Ca and by the pnQRPA for ^{76}Ge. The short-range correlations have been calculated by the use of the simple Jastrow function and the more refined UCOM method. Our computed results indicate that the Jastrow method cuts off relevant parts of the many-body wave function for high values of angular momentum of the intermediate states. This leads to the excessive reduction of 30%–40% in the magnitudes of the nuclear matrix elements. At the same time the UCOM reduces the magnitudes of the matrix elements only by 7%–16%, roughly equally for all multipoles. Our results put to question the recent calculations where short-range and tensor correlations cause large effects on the nuclear matrix elements of neutrinoless double beta decay [7]. Study of the effects of the UCOM procedure upon heavier nuclei is in progress.

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